STRATEGIC CHARACTERIZATION OF RISK SYSTEMS

Dr. habil István BUKOVICS CSc

Az 1960-as évektől a kockázati rendszerek (KR) tanulmányozása fokozott intenzitással folyik. Közülük valószínűleg a legaktuálisabbak az extrém klímahatások fenyegetéseivel foglalkozó KR-ek. Ezek alakulásának bemutatására, modellezésére alkalmas a hibafa módszer, amely indirekt monoton növekvő Boole függvényeket használ.

1. Introduction

Risk systems (RS) are studied with growing intensity from the 1960's[1]. Among them one of the most timely is probably that of threatened by the climatic extremities. Their behavior can be both represented and modeled by fault tree methodology that uses indirect monotone increasing (a.k.a. positive) Boolean functions. These functions have typically about a hundred Boolean variables called "basic events" that we prefer to call "Primary events" (*prime even*ts for short). The *states* of RS are represented by these prime events, p1, p2,…,pn, n integer, fixed. Let, from now on, the indirect Boolean function ("Fault Tree") describing RS be denoted by FT(p1, p2,…,pn) in a direct form or, equivalently, in the indirect form as FT(f, g,...), with f, g,... are again indirect Boolean functions of p1, p2,…,pn.

FT being positive, negated variables never occurs in it, moreover (by the explication[2] of FT) each f, g,.. is either a pure conjunction or disjunction of p1, p2,…,pn or other positive indirect Boolean functions of p1, p2,…,pn.

In this work we slightly expand that theory by using *ternary* logic instead of binary for the sake of easier interpretation[3]. So, for now on, RS will be described (modeled) by a *ternary* indirect monotonic function FT(p1, p2,…,pn), n integer, fixed, where each pi (i = 1, 2,…,n) is a ternary variable with values 0, u,1. These are interpreted respectively as

pi = 0 whenever the prime event (belonging to pi) does not occur (i.e. is not the case),

pi = 1 whenever the prime event (belonging to pi) does occur (i.e. is the case).

pi = u whenever the prime event (belonging to pi) is "*undefended*". This – as the "third logical value" - is interpreted within traditional ternary logic as "uncertain" or "undetermined" or "unknown". If pi = u then pi is called a **free prime** (event).

As in Boolean logic we define an *ordering* relationship postulating 0 < u < 1. By this, we define c*onjunction* and *disjunction* as

$p \wedge q = \min(p, q)$ and $p \vee q = \max(p, q)$

respectively for arbitrary ternary variables p, q.

---

[1] See e.g. [Henley - Kumamoto]

[2] A very thorough discussion of the important notion of explication in general can be found in Carnap

[3] See e.g. Jorge Pedraza Arpasi .A Brief Introduction to Ternary Logic. 7th November 2003. http://www.aymara.org/ternary/ternary.pdf

2. Preliminaries

Evidently, for arbitrary ternary variables p, q, r the following axioms for the *distributive lattices* hold:

$p \wedge (q \wedge r) = (p \wedge q) \wedge r$ and $p \vee (q \vee r) = (p \vee q) \vee r$

$p \wedge q = q \wedge p$ and $p \vee q = q \vee p$

$p \wedge (q \vee p) = p$ and $p \vee (q \wedge p) = p$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ and $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

In addition::

$p \wedge q = p$ if and only if $p \vee q = q$

$p \leq q$ if and only if $p \wedge q = p$

Now let any series p1, p2,…,pn be denoted by $\underline{\mathbf{p}}$ called a “*state vector*”.

- If for a $\underline{\mathbf{p}}$, FT($\underline{\mathbf{p}}$) = 1 then we say that the risk system, described by the ternary indirect function FT is *active* in the state $\underline{\mathbf{p}}$.
- If for a $\underline{\mathbf{p}}$, FT($\underline{\mathbf{p}}$) = 0 then we say that the risk system, described by the ternary indirect function FT is *passive* in the state $\underline{\mathbf{p}}$.
- If for a $\underline{\mathbf{p}}$, FT($\underline{\mathbf{p}}$) = x then we say that the risk system, described by the ternary indirect function FT is *undetermined or free* in the state $\underline{\mathbf{p}}$.

For any state vectors $\underline{\mathbf{p}}$, $\underline{\mathbf{q}}$ we define

$\underline{\mathbf{p}} \leq \underline{\mathbf{q}}$ if and only if for all i = 1,…,n pi ≤ qi,

$\underline{\mathbf{p}} \geq \underline{\mathbf{q}}$ if and only if $\underline{\mathbf{q}} \leq \underline{\mathbf{p}}$

and

$\underline{\mathbf{p}} < \underline{\mathbf{q}}$ if and only if $\underline{\mathbf{p}} \leq \underline{\mathbf{q}}$ and $\underline{\mathbf{p}} \neq \underline{\mathbf{q}}$

It follows form the above:

$\underline{\mathbf{p}} \leq \underline{\mathbf{q}}$ and $\underline{\mathbf{q}} \leq \underline{\mathbf{r}}$ implies $\underline{\mathbf{p}} \leq \underline{\mathbf{r}}$

$\underline{\mathbf{p}} \leq \underline{\mathbf{q}}$ and $\underline{\mathbf{q}} \leq \underline{\mathbf{p}}$ implies $\underline{\mathbf{p}} = \underline{\mathbf{q}}$

$\underline{\mathbf{p}} \leq \underline{\mathbf{q}}$ and $\underline{\mathbf{p}} = \underline{\mathbf{q}}$ implies $\underline{\mathbf{p}} = \underline{\mathbf{q}}$ or $\underline{\mathbf{p}} < \underline{\mathbf{q}}$

Using this terminology we say, that a ternary function *is monotone increasing* if always:

$\underline{\mathbf{p}} \leq \underline{\mathbf{q}}$ implies FT($\underline{\mathbf{p}}$) ≤ FT($\underline{\mathbf{q}}$)

In practice, the only way to influence a risk system’s behavior i.e. the value of FT, is to assign values to the state vector $\underline{\mathbf{p}}$. To be more precise, it can be supposed that in practice in many cases there are at our disposal to apply certain *operators* to any primary state.

One of the two to be introduced is called henceforward “*Passivators*” $\underline{\mathbf{P}}$1, $\underline{\mathbf{P}}$2,…, $\underline{\mathbf{P}}$n with

$\underline{\mathbf{P}}$i (p1,…,p(i-1), u, p(i +1),…,pn) = (p1,…, p(i-1), 0, p(i +1),…,pn)

That is, applying a passivator $\underline{\mathbf{P}}$i to a state $\underline{\mathbf{p}}$, one (the “**Defender**” of the risk system in question) is able to *passivate* the i-th free prime .

On the other hand, the defender of RS must face the "**Attacke**r" of the RS. The attacker can be (interpreted as) a terrorist (attack) a saboteur (action) or the Nature (the Environment) itself or the like.
The attacker can be modeled by a collection of operators **A**i (called "Prime *Activators*") with

Ai (p1,…,p(i-1), u, p(i +1),…,pn) = (p1,…, p(i-1), 1, p(i +1),…,pn)

In words: applying an activator **A**i to a state **p**, one (the "attacker" of the risk system in question) is able to activate the i-th free prime.
Within this model a *strategic game* – let it be called a "**Shannon game"** – can be defined where:

- The number of **players** is two: "Attacker" and "Defender","A" and "D" for short).
- The possible **steps** for the Attacker (**A-Step** for short) to passivate any free prime i.e. to perform the transition pi = u →pi = 1 for some i = 1,…,n.
- The possible **steps** for the Defender (**D-Step** for short) to passivate any free prime i.e. to perform the transition pi = u →pi = 0 for some i = 1,…,n.
- The **payment** for an A_step is the "renovation cost" and / or "renovation time" (RenCost and RenTime for short respectively).
- The **payment** for a D_step is the "prevention cost" and / or "prevention time" (PrevCost and PrevTime for short respectively)
- **The rules of the game**:
  (1) the first step is made by **A,**
  (2) each A-step consist of randomly passivating a free prime,
  (3) the D-steps varies according to the *defense algorithms* or *defense strategies* that the Defender follows or combines.
  These will be detailed later in the paper.
  (4) the players follow each other alternatively.
  (5) the **end of the game** is the case, when the top event is either true or false (i.e. not uncertain). In the first case **A**, else **D** is the winner.

3. The defense strategies

The best way for grasping the intuitive meaning of the defense strategy of RS is to consider an example represented by a customary fault tree with the top event: "NITROGEN TO WATERSHED" due to [**Buck**]. The fault tree is represented by us in a windows's explorer-like fashion, the details of which can be seen below on Figure 1 and 2.

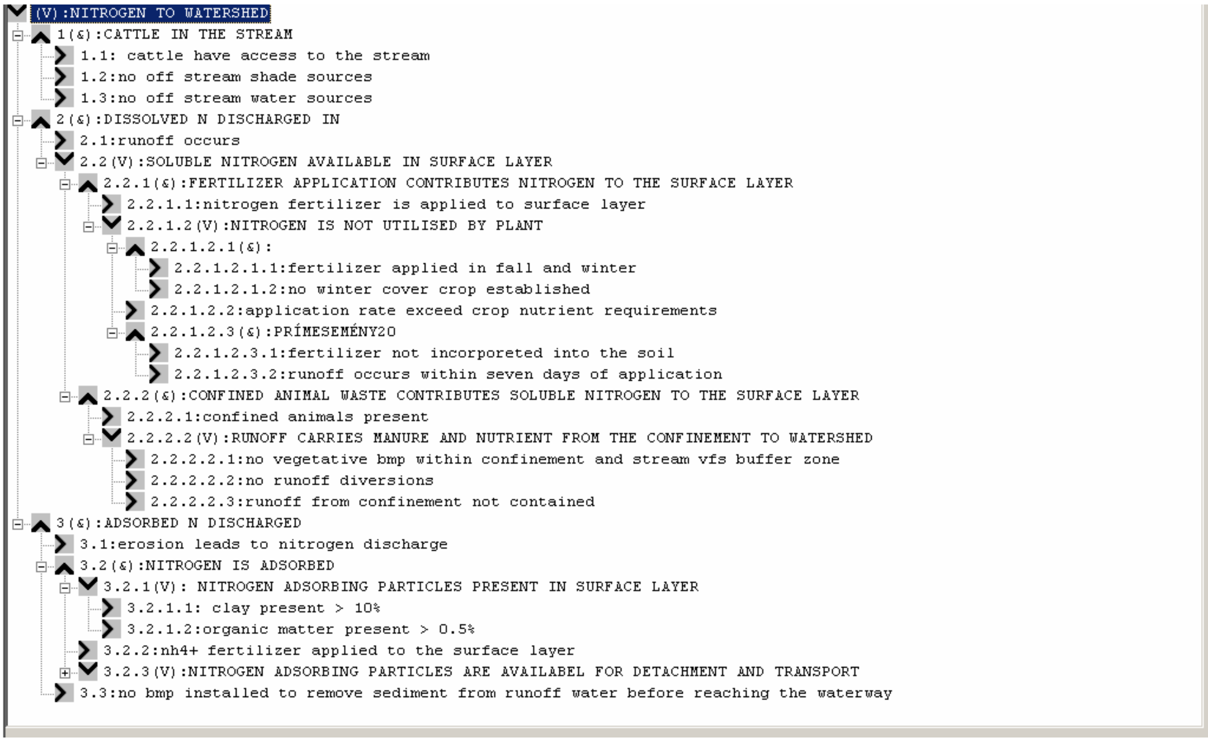

Figure 1.
Details of a fault tree in an expandable form (to avoid the obsolete and clumsy gate drawing techique). The disjunctive composite event "3.2.3(V):NITROGEN ADSORBING PARTICLES ARE AVAILABLE FOR DETACHMENT AND TRANSPORT" can be further expanded, see Figure 2.

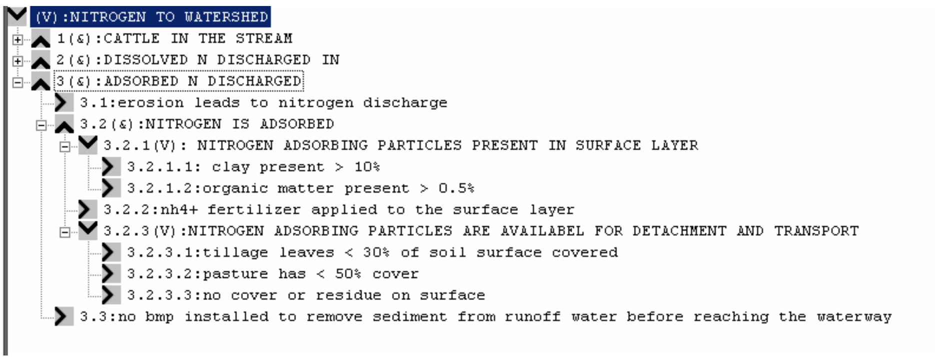

Figure 2
The fault tree if Figure 1 with the disjunctive event ("OR Gate") 3.2.3(V) expanded.

4. Some examples of a strategy

Suppose, that the two players (A and D) play a game using the strategy called "Struggling".

The rule of this game is simply choosing randomly any free prime and acting. Suppose, that at the beginning (of a thought experiment, or model-game) every prime event is undefended, (i.e. *free*) and the Attacker randomly chooses and activates a free prime event, say p2. The Defender's response is p22 and so on. See Table 1. for details[4].

Table 1

```
The Top Event of the Risk System: (V):NITROGEN TO WATERSHED
Reference:
"Excessive nitrogen is discharged to Owl Run on any day",
scholar.lib.vt.edu/theses/available/etd-17391653976940/unrestricted/etd.pdf

The Strategy: Struggling Strategy
Number of Prime Events = 22
Number of Complex Events = 36
The winner of the game: A (Attacker)
Number of steps = 22
LEGEND:
n0: the number of the passivated prime events
n1: the number of the activated prime events
nU: the number of the undefended (free) prime events
A: `Attacker` (`Activator`)
D: `Defender` (`Passivator`)
The four `Franklin parameters`: Prevention/Renovation Cost/Time respectively
are measured by an arbitrary (but fixed in advance) unit, say 100% = 100 USA
Dollar.
(The actual data are randomly determined for the sake of the example.)
```

STATE BEFORE STEP	THE PLAYER	THE STEP	PREVENTION COST [%]	PREVENTION TIME [%]	RENOVATION COST [%]	RENOVATION TIME [%]	STATE AFTER STEP	Row
n0 = 0, n1 = 0, nU = 22	A:	p2 --> 1			96	54	n0 = 0, n1 = 1, nU = 21	01
n0 = 0, n1 = 1, nU = 21	D:	p22 --> 0	52	82			n0 = 1, n1 = 1, nU = 20	02
n0 = 1, n1 = 1, nU = 20	A:	p14 --> 1			72	70	n0 = 1, n1 = 2, nU = 19	03
n0 = 1, n1 = 2, nU = 19	D:	p21 --> 0	62	33			n0 = 2, n1 = 2, nU = 18	04
n0 = 2,	A:	p17 -->			84	39	n0 = 1,	05

[4] Table 1 is due to Profes LTD, www.profes.hu

n1 = 2, nU = 18		1					n1 = 3, nU = 18	
n0 = 1, n1 = 3, nU = 18	D:	p12 --> 0	41	79			n0 = 2, n1 = 3, nU = 17	06
n0 = 2, n1 = 3, nU = 17	A:	p19 --> 1			18	58	n0 = 2, n1 = 4, nU = 16	07
n0 = 2, n1 = 4, nU = 16	D:	p3 --> 0	76	60			n0 = 3, n1 = 4, nU = 15	08
n0 = 3, n1 = 4, nU = 15	A:	p10 --> 1			01	11	n0 = 3, n1 = 5, nU = 14	09
n0 = 3, n1 = 5, nU = 14	D:	p9 --> 0	24	48			n0 = 4, n1 = 5, nU = 13	10
n0 = 4, n1 = 5, nU = 13	A:	p6 --> 1			94	14	n0 = 4, n1 = 6, nU = 12	11
n0 = 4, n1 = 6, nU = 12	D:	p15 --> 0	80	88			n0 = 5, n1 = 6, nU = 11	12
n0 = 5, n1 = 6, nU = 11	A:	p5 --> 1			23	77	n0 = 5, n1 = 7, nU = 10	13
n0 = 5, n1 = 7, nU = 10	D:	p16 --> 0	19	29			n0 = 6, n1 = 7, nU = 9	14
n0 = 6, n1 = 7, nU = 9	A:	p13 --> 1			95	12	n0 = 6, n1 = 8, nU = 8	15
			354	419	483	335		

As an other example of the Struggling strategy, consider the case of Hayek, Table 2

Table 2

The Top Event of the Risk System: (&):HT WEED
Reference:
Keith R. Hayes: Final report: Inductive hazard analysis for GMOs
CSIRO Division of Marine Research 2004
//www.deh.gov.au/settlements/publications/biotechnology/hazard/fault.html

The Strategy: Struggling Strategy
Number of Prime Events = 111
Number of Complex Events = 167
The winner of the game: D (Defender)
Number of steps = 9
LEGEND:
n0: the number of the passivated prime events
n1: the number of the activated prime events
nU: the number of the undefended (free) prime events
A: \`Attacker\` (\`Activator\`)
D: \`Defender\` (\`Passivator\`)
The four \`Franklin parameters\`: Prevention/Renovation Cost/Time respectively

are measured by an arbitrary (but fixed in advance) unit, say 100% = 100 USA Dollar.
(The actual data are randomly determined for the sake of the example.)

STATE BEFORE STEP	THE PLAYER	THE STEP	PREVENTION COST [%]	PREVENTION TIME [%]	RENOVATION COST [%]	RENOVATION TIME [%]	STATE AFTER STEP	Row
n0 = 0, n1 = 0, nU = 111	A:	p41 --> 1			52	75	n0 = 0, n1 = 1, nU = 110	001
n0 = 0, n1 = 1, nU = 110	D:	p62 --> 0	63	24			n0 = 1, n1 = 1, nU = 109	002
n0 = 1, n1 = 1, nU = 109	A:	p53 --> 1			34	31	n0 = 1, n1 = 2, nU = 108	003
n0 = 1, n1 = 2, nU = 108	D:	p14 --> 0	89	97			n0 = 2, n1 = 2, nU = 107	004
n0 = 2, n1 = 2, nU = 107	A:	p105 --> 1			88	04	n0 = 2, n1 = 3, nU = 106	005
n0 = 2, n1 = 3, nU = 106	D:	p4 --> 0	79	33			n0 = 3, n1 = 3, nU = 105	006
			231	154	174	110		

5. Other strategies

The Struggling strategy is completely independent from the financial circumstances of the RS. Its success depends only on the logic structure of the RS. At the same time, however, it does not refer explicitly to it. It is desirable to invent a strategy that adapts itself to the actual form of the RS. In next part of the paper, we attempt to develop such a strategy. As an honour to Claude Shannon we name it “Shannon Strategy”.
In practice, every defence step is heavily influenced from the financial circumstances. To take these circumstances into consideration, we offer the following four indicators, called Franklin-parameters[5].

- The prevention cost
- The prevention time
- The renovation cost
- The renovation time

Each defence step has its own price: do defend an event (either prime or complex) it takes some cost and time to prevent it. Similarly, each attack entails some renovation cost and time. It is quite natural to define strategies aiming at to minimize the prevention cost viz. time of the prime events in question. In other words, the defence strategy “**MinPrevCost**” means to choose the free prime to defend with the minimal prevention cost. The other defence strategy “**MinPrevTime**” can be defined in an analogue way.
Mathematically the “**MinRenCost**” and “**MinRenTime**” is quite similarly treatable, but their interpretation is somewhat problematic. But if the attacker is intelligent that it will prefer to attack the primes with the maximal renovation cost or time. However just these are defended, thus the attacker will be deprived from causing the biggest damage. The situation is vaguely similar to the “secondary change” thoroughly

[5] Cf. Table 1 and 2.

discussed in Watzlawick[6] An other kind of strategy, that is close to what is known as *maintenance*, is the way one exchanges (typically mechanical) components e.g. in process plants if their reliability (due to, say, fatigue) is under a predetermined minimum. This strategy will be called "**AdHoc**"
Now the question naturally arises whether a risk system behaves differently or not against different strategies. If it does, one can determine the best one. If not, one can classify and characterize RS by their very behaviour wrt strategies.

References

Buck, Sharon P.:
Applying Probabilistic Risk Assessment to Agricultural Nonpoint Source Pollution.
MSc Thesis, State University, Virginia, 1997.
(http://scholar.lib.vt.edu/theses/available/etd-17391653976940/unrestricted/etd.pdf)

Carnap, R.: Logical Foundations of Probability. The University of Chicago Press, Routledge & Kegan PaulLondon, 1963, pp. 1-18

Henley, E. J., - Kumamoto, H.: Reliability Engineering and Risk Assessment
Prentice Hall, 1981.

Watzlawick, Paul, John H. Weakland, Richard Fisch
Change: Principles of Problem Formation and Problem Resolution
W W Norton & Company

[6] See Waclawick. An ad says: "A provocative view of the process of change, informed by a clinical perspective. Three prominent American therapists detail their theories and strategies for promoting human change and dealing with related psychological problems."
(http://www.mri.org/bookone.html#change)